THE QUANTUM EXPLORATION OF SPACE-TIME
- HOLOMETER -
FERMI NATIONAL ACCELERATOR LABORATORY

2011/08/05

Holometer User's Manual

Interferometer Algebra

Douglas Kim
Cornell University, Fermi National Accelerator Laboratory

Distribution of this document:
all

This is an internal working note of the Holometer project

Fermi National Accelerator Laboratory

E-mail: holo@fnal.gov
`http://holometer.fnal.gov/`

Abstract

Holographic theory suggests that physics changes radically at the Planck length, $l_p = \sqrt{\hbar G/c^3} = 1.6 \times 10^{-35} m$. Black hole and string theories theorize that at such small length scales the universe is holographic, with reality existing in two dimensions with the third dimension conjointly linked with time. The Fermilab Holometer is an experiment designed to investigate the nature of space-time on these Planckian length scales. The Holometer is an extremely sensitive laser interferometer that, upon completion, will surpass the sensitivity of the GEO600 and LIGO gravitational wave detectors. At such high sensitivities, the Holometer should detect holographic fluctuations in space-time. The experiment uses two power-recycled interferometers in close proximity of their causal space-time diamonds in order to see correlated holographic noise. In order to understand the power response of the power-recycled interferometers, the equations for the electric fields within the interferometers were derived. These derivations can be understood and utilized in a high school setting being that the highest order of math needed is an algebra background with knowledge in basic complex numbers. With these derivations, one can also investigate various properties of the interferometer such as the power output at the antisymmetric port, the finesse of the lasers, and power spectrum of the holographic noise.

This paper is part of a larger collaborative manual that will provide knowledge on holographic theory and the apparatus in an accessible manner for all who are interested. This manual is still a work in progress.

1 Introduction

Max Karl Ernst Ludwig Planck in 1899 proposed a set of five fundamental units of measure defined by five universal constants: the gravitational constant G, reduced Planck constant $\hbar$, speed of light c, Coulomb constant k, and Boltzmann's constant k_b. Simultaneously solving the equations describing gravitational force, mechanical force, thermal energy, Coulomb's law, and momentum in terms of Planck units, one can derive the Planck time, length, mass, charge, and temperature. The Planck length, $l_p = \sqrt{\hbar G/c^3} = 1.6 \times 10^{-35} m$, is of particular interest to physicists. It is theorized that the physics of space and time change drastically at such small length scales.

2 Theory

2.1 The Entropy of Information

So why is the Planck length of such interest? In order to answer this question, we must delve into the realm of information theory and black hole physics. Information has become a ubiquitous part of our everyday lives. For example, the systems and processes that handle and store information dominate the Internet. The technology that advances the storage size and transfer rate of information improves exponentially year by year. But one must ask, when will this progress stop? Is there such thing as a bandwidth limit for information? For the universe?

To answer this question we turn to statistical mechanics and the concept of entropy. Entropy is defined as a measure of the amount of energy that is available to do work. Quantitatively, entropy is a state variable, which is some measurable property of an object or system, that is defined for a reversible thermodynamic process at some temperature $T$and heat Q. Similarly in information theory, the entropy of a communications system, or the Shannon entropy, is a measure of the amount of information in a system.

Conceptually, thermodynamic entropy and Shannon entropy are equivalent. The difference is a matter of convention, in particular the modes of freedom considered. For example, the Shannon entropy of a transistor only depends on the overall state of the transistor, which is either on or off, 0 or 1. Therefore, there are only two degrees of freedom in this system. On the contrary, the thermodynamic entropy of the same transistor depends on all of the states of the atoms that make up the transistor. It is calculated for a single silicon microchip that the Shannon entropy is 1010 bits, whereas the thermodynamic entropy is about 1023 bits at room temperature [3]. However, when calculating the entropies with the same degrees of freedom, the Shannon and thermodynamic entropies are the same. This leads us to several questions. What is the maximum degree of freedoms for any system? What is the maximum number of information we can fit into a system? What is the bandwidth limit of the universe?

This is where we are at the limits of our knowledge. In order to understand the information capacity of a system, we must understand the system at its smallest limit of structure. This limit is the Planck length. But another question arises. Why is the limit the Planck length and not some other arbitrary length? In order to answer this question, we must go into the realm of black hole thermodynamics.

2.2 Black Holes and the Second Law of Thermodynamics

In 1915, Albert Einstein published his theory of general relativity. General relativity unifies special relativity and Newtonian gravitation by describing gravity as a property of space-time. The theory suggests that the curvature of space-time is dependent on the mass-energy and linear momentum of the present matter and radiation. The interaction of gravity and space-time are described by the Einstein field equations, a set of ten partial differential equations. A couple of months later, Karl Schwarzschild obtained an exact solution to these equations. His solution had a peculiar behavior at what is known as the Schwarzschild radius, where certain terms of the Einstein field equations blew up to infinity creating a singularity.

This singularity implies the existence of black holes.

A property of a black hole is the event horizon, which is a boundary of space-time. If matter or light passes the event horizon, they can only go inwards toward the mass of the black hole. Once over the event horizon, nothing, including light, can escape the gravity of the black hole. If an event happened within the event horizon, an observer would never know if the event ever occurred. Thus, information, once over the event horizon, can never reach an observer on the outside of the horizon.

The laws of conservation of energy and angular momentum are conserved in black holes. This can be verified through the interaction of black holes with the surrounding space-time as it collects angular momentum and mass from matter that falls into its event horizon. However, it appears that black holes violate the second law of thermodynamics.

The second law of thermodynamics qualitatively states that most processes in nature are irreversible. For example, after you burn a log for fire, it is highly unlikely that the log will rematerialize. When matter gets sucked into a black hole, it seems as though the entropy disappears too, which violates the second law of thermodynamics. To reconcile this violation, Stephen Hawking and Jacob Bekenstein theorized that the entropy of a black hole is proportional to the area of the event horizon of the black holes event horizon divided by the Planck area, $S_{BH} = kA/4l_p^2$.

2.3 We Are . . . Holograms?!

So now let's make a connection between the thermodynamic entropy of a black hole to its Shannon entropy. Shannon entropy is measured in bits and the total quantity of bits is determined by the total degrees of freedom in the system. How dense we can package information into space-time is determined by these degrees of freedom. Based on the entropy of a black hole and AdS/CFT correspondence theories, the limit of information we can pack into space-time is the Planck area constrained to 2+1 dimensions. What does this mean? This means that we are living in a holographic universe. But what does it mean that the universe is holographic? It means that our perception of reality as three-dimensional is an illusion. We are actually two-dimensional entities with the third dimension inextricably linked with time.

You might be unconvinced that you are a hologram. However, this hypothesis can be investigated through an experiment such as the Fermilab Holometer. But how would you figure out if you were a hologram or not? The holographic principle says that there is a bandwidth limit on the universe. This is analogous to the statement that space-time is pixelated at the

Planck scale. One way to approach this idea is to imagine the universe as a movie. If you were a character in a movie and took a high-powered microscope and looked closely enough, you would discover that you are made of blue, red, and green pixels. Similarly with the Holometer, we are looking at an extremely small length scale, the Planck length. Sadly, a microscope with Planckian resolution does not exist. However, using interferometers, we can investigate the physics of space-time at the Planck scale.

3 Experimental Setup

The Holometer consists of two power-recycled Michelson interferometers in close proximity such that their causal light diamonds overlap. An interferometer is a device that superimposes electromagnetic waves in order to gather information on the waves. This is usually done by splitting and recombining the waves through a beam splitter. The resulting inference pattern is determined by the phase differences of the recombined waves. Waves in phase will result in constructive inference patterns, whereas waves out of phase result in destructive patterns.

A Michelson interferometer uses a single laser beam splitter for separating and recombining electromagnetic waves. A coherent light source (in our case a laser beam) is incident on a 50/50 beam splitter which splits the beam off into two perpendicular optical arms. The beams reflect off highly reflective mirrors and recombine at the beam splitter. The recombined beam, called the antisymmetric beam, is then analyzed at a photodetector.

The vibrations in space-time due to holographic noise lead to fluctuations in the measured phase of the laser beam, which drift about a Planck length per Planck time. Two interferometers in close proximity of each other should see the same correlated holographic signal.

4 Field Calculations for a Power-Recycled Michelson Interferometer

4.1 The 1D Optical Cavity

Electromagnetic waves propagate in space as $E = E_0 e^{ikl}$, where E_0 is the initial wave amplitude, k is the wavenumber, and l is the length of the optical cavity. In an interferometer, the length of the optical cavities determines the phase of the wave. In order to simplify the calculations, we will consider the scattering states using the complex symmetric form $S = [r, it, it, r]$ as suggested in *Lasers* by Siegman [1, p. 406]. Remembering this symmetric form is easier than using the physical bound conditions of the mirrors and beam splitter. This means that a plane wave hitting a mirror with reflectivity r and transmission t produces a reflected beam with amplitude $E_R = rE_0$ and a transmitted beam with amplitude $E_T = itE_0$. Using a convention where we have to know the physical boundary conditions would require us to determine whether or not the reflected coefficient is negative or positive. It is much easier to remember that the reflective coefficient is simply r and that the transmitted coefficient is it. In general, a mirror will also have a loss a, where $r^2+t^2+a^2 = 1$.

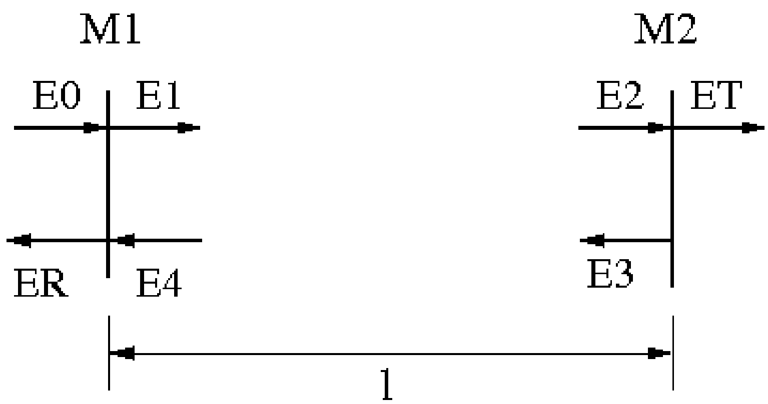

Figure 1: 1D optical cavity

The one-dimensional optical cavity in Figure 1 has an input beam of amplitude E_0 to two mirrors (M1 and M2). One of the things we need to do is calculate the circulated (E_1), reflected (E_R), and transmitted (E_T) amplitudes in the optical cavity. Deriving these equations will help us calculate and measure the correlations in the optical phase flucuations of the antisymmetric ports (the power of E_T) of the two Michelson interferometers [2]. In order to do this, the wave amplitudes must first be determined. The wave amplitude can either be described as a wave propagation in space-time like equations 1b and 1d, or a superposition of reflected and transmitted beam amplitudes like equations 1a, 1c, 1e, or 1f. If the wave has propagated the length of the optical cavity, its wave amplitude is described as $E = E_0e^{ikl}$ with a collected phase of kl (l being the distance that the wave traveled in the cavity) . For example, E_2 is the resultant propagation of E_1, so its wave amplitude is simply $E_2 = E_1e^{ikl}$.

On the other hand, E_1 is formed by the transmitted amplitude of E_0 and the reflected amplitude of E_4; therefore the wave amplitude for E_1 is $E_1 = it_1E_0 + r_1E_4$. As another example, E_3 is only the reflected beam of E_2, therefore its wave amplitude is simply $E_3 =$

$r_2 E_2$. Using this method, one can describe all the wave amplitudes in a 1D cavity.

$$E_1 = it_1 E_0 + r_1 E_4 \tag{1a}$$
$$E_2 = E_1 e^{ikl} \tag{1b}$$
$$E_3 = r_2 E_2 \tag{1c}$$
$$E_4 = E_3 e^{ikl} \tag{1d}$$
$$E_R = r_1 E_0 + it_1 E_4 \tag{1e}$$
$$E_T = it_2 E_2 \tag{1f}$$

Now we can use a scatter matrix and row reduction to solve for E_1, E_R, and E_T simultaneously. However, the three expressions can be solved directly using substitution. We shall use the latter method in that it requires no knowledge of linear algebra.

Solving for the circulated beam:

$$E_1 = it_1 E_0 + r1 E_4 \tag{2a}$$
$$= it_1 E_0 + r_1 e^{ikl} \tag{2b}$$
$$= it_1 E_0 + r_1 r_2 e^{ikl} \tag{2c}$$
$$= it_1 E_0 + r_1 r_2 E_1 e^{2ikl} \tag{2d}$$
$$E_1 - r_1 r_2 e^{2ikl} = it_1 E_0 \tag{2e}$$
$$E_1(1 - r_1 r_2 e^{2ikl}) = it_1 E_0 \tag{2f}$$
$$\tag{2g}$$

and thus

$$\frac{E_1}{E_0} = \frac{it_1}{1 - r_1 r_2 e^{2ikl}} \tag{3}$$

Solving for the transmitted beam:

$$E_T = it_2 E_2 \tag{4a}$$
$$= it_2 e^{ikl} E_1 \tag{4b}$$
$$= \frac{-t_1 t_1 e^{ikl} E_0}{1 - r_1 r_2 e^{2ikl}} \tag{4c}$$

so

$$\frac{E_T}{E_0} = \frac{-t_1 t_2}{e^{-ikl} - r_1 r_2 e^{ikl}}. \tag{5}$$

At resonance ($e^{ikl} = 1$), for lossless mirrors ($r^2 + t^2 = 1$) with identical reflectivity, the ratio reduces down to 1.

Solving the reflected beam:

$$
\begin{aligned}
E_R &= r_1 E_0 + it_1 E_4 && \text{(6a)}\\
&= r_1 E_0 + it_1 e^{ikl} E_3 && \text{(6b)}\\
&= r_1 E_0 + it_1 r_2 e^{ikl} E_2 && \text{(6c)}\\
&= r_1 E_0 + it_1 r_2 e^{2ikl} E_1 && \text{(6d)}\\
&= r_1 E_0 - \frac{t_1^2 r_2 e^{2ikl} E_0}{1 - r_1 r_2 e^{2ikl}} && \text{(6e)}
\end{aligned}
$$

so

$$
\frac{E_R}{E_0} = r_1 - \frac{t_1^2 r_2 e^{2ikl}}{1 - r_1 r_2 e^{2ikl}} \tag{7}
$$

At resonance ($e^{ikl} = 1$), for lossless mirrors ($r^2 + t^2 = 1$) with identical reflectivity, the ratio reduces to 0.

For lossless mirrors, with $r_1 = 0.95$ and $r_2 = 0.99999$, the transmitted power is

$$
P_t = (E_T/E_0)^2 = 7.217 \times 10^{-3} \tag{8}
$$

The power incident on the cavity equals the power reflected plus the power transmitted.

$$
E_0^2 = E_T^2 + E_R^2 \tag{9}
$$

This relation holds true for the special case where the two mirrors are the same, with no loss ($a_1 = a_2 = 0$) and $r_1 = r_2 = r$, $t_1 = t_2 = t$, and $r^2 + t^2 = 1$.

It can be shown that for the general case where $r_1 \neq r_2$ and $e^{ikl} = 1$ that power is conserved.

$$
\begin{aligned}
E_0^2 &= E_T^2 + E_R^2 && \text{(10a)}\\
1 &= (E_T/E_0)^2 + (E_R/E_0)^2 && \text{(10b)}\\
&= \left(\frac{-t_1 t_2}{e^{-ikl} - r_1 r_2 e^{i\phi}}\right)^2 + \left(r_1 - \frac{t_1^2 r_2 e^{2ikl}}{1 - r_1 r_2 e^{2ikl}}\right)^2 && \text{(10c)}\\
&= \frac{(1-r_1^2)(1-r_2^2)}{(1-r_1 r_2)^2} + \frac{(r_1(1-r_1 r_2) - r_2(1-r_1^2))^2}{(1-r_1 r_2)^2} && \text{(10d)}\\
&= \frac{1 - r_2^2 - r_1^2 + r_1^2 r_2^2}{1 - 2r_1 r_2 + r_1^2 r_2^2} + \frac{(r_1 - r_2)^2}{1 - 2r_1 r_2 + r_1^2 r_2^2} && \text{(10e)}\\
&= \frac{(1 - r_2^2 - r_1^2 + r_1^2 r_2^2) + (r_1^2 - 2r_1 r_2 + r_2^2)}{1 - 2r_1 r_2 + r_1^2 r_2^2} && \text{(10f)}\\
&= \frac{1 - 2r_1 r_2 + r_1^2 r_2^2}{1 - 2r_1 r_2 + r_1^2 r_2^2} && \text{(10g)}\\
1 &= 1 && \text{(10h)}
\end{aligned}
$$

4.2 The 2D Optical Cavity

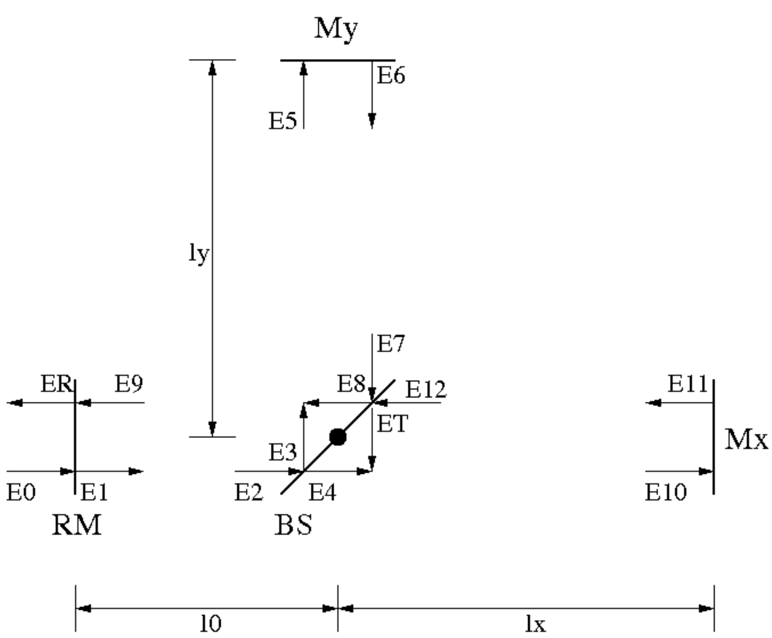

Figure 2: 2D optical cavity

The wave amplitudes for the 2D cavity are derived in the same fashion as the 1D cavity.

The only difference is that now we have a beam splitter and three 1D cavities. Using the same method, we determine the wave amplitudes to be the following:

$$E_1 = it_{rm}E_0 + r_{rm}E_9 \tag{11a}$$
$$E_2 = E_1e^{ikl_0} \tag{11b}$$
$$E_3 = r_{bs}E_2 \tag{11c}$$
$$E_4 = it_{bs}E_2 \tag{11d}$$
$$E_5 = E_3e^{ikl_y} \tag{11e}$$
$$E_6 = r_yE_5 \tag{11f}$$
$$E_7 = E_6e^{ikl_y} \tag{11g}$$
$$E_8 = r_{bs}E_7 + it_{bs}E_{12} \tag{11h}$$
$$E_9 = E_8e^{ikl_0} \tag{11i}$$
$$E_{10} = E_4e^{ikl_x} \tag{11j}$$
$$E_{11} = r_xE_{10} \tag{11k}$$
$$E_{12} = E_{11}e^{ikl_x} \tag{11l}$$
$$E_R = r_mE_0 + it_mE_9 \tag{11m}$$
$$E_T = r_{bs}E_{12} + it_{bs}E_7 \tag{11n}$$

And again, using direct substitution we solve for E_1, E_R, and E_T.

Solving for the circulated beam:

$$E_1 = it_mE_0 + r_mE_9 \tag{12a}$$
$$= it_mE_0 + r_m(E_8e^{ikl_0}) \tag{12b}$$
$$= it_mE_0 + r_me^{ikl_0}(r_{bs}E_7 + it_{bs}E_{12}) \tag{12c}$$
$$= it_mE_0 + r_me^{ikl_0}(r_{bs}e^{ikl_y}E_6 + it_{bs}e^{ikl_x}E_{11}) \tag{12d}$$
$$= it_mE_0 + r_me^{ikl_0}(r_{bs}r_ye^{ikl_y}E_5 + it_{bs}r_xe^{ikl_x}E_{10}) \tag{12e}$$
$$= it_mE_0 + r_me^{ikl_0}(r_{bs}r_ye^{2ikl_y}E_3 + it_{bs}r_xe^{2ikl_x}E_4) \tag{12f}$$
$$= it_mE_0 + r_me^{ikl_0}(r_{bs}^2r_ye^{2ikl_y}E_2 - t_{bs}^2r_xe^{2ikl_x}E_2) \tag{12g}$$
$$= it_mE_0 + r_me^{ikl_0}(r_{bs}^2r_ye^{2ikl_y}e^{ikl_0}E_0 - t_{bs}^2r_xe^{2ikl_x}e^{ikl_0}) \tag{12h}$$

$$\frac{E_1}{E_0} = \frac{it_m}{1 - r_me^{2ikl_0}(r_{bs}^2r_ye^{2ikl_y} - t_{bs}^2r_xe^{2ikl_x})} \tag{13}$$

Solving for the transmitted beam:

$$
\begin{aligned}
E_T &= r_{bs}E_{12} + it_{bs}E_7 && \text{(14a)}\\
&= r_{bs}e^{ikl_x}E_{11} + itbse^{ikl_y}E_6 && \text{(14b)}\\
&= r_{bs}r_xe^{ikl_x}E_{10} + it_{bs}r_ye^{ikl_y}E_5 && \text{(14c)}\\
&= r_{bs}r_xe^{2ikl_x}E_4 + it_{bs}r_ye^{2ikl_y}E_3 && \text{(14d)}\\
&= it_{bs}r_{bs}r_xe^{2ikl_x}E_2 + it_{bs}r_{bs}r_ye^{2ikl_y}E_2 && \text{(14e)}\\
&= it_{bs}r_{bs}r_xe^{2ikl_x}e^{ikl_0}E_1 + it_{bs}r_yr_{bs}e^{2ikl_y}e^{ikl_0}E_1 && \text{(14f)}\\
& && \text{(14g)}
\end{aligned}
$$

Therefore:

$$
\frac{E_T}{E_0} = \frac{-t_me^{ikl_0}(t_{bs}r_{bs}r_xe^{2ikl_x} + t_{bs}r_{bs}r_ye^{2ikl_y})}{1 - r_me^{2ikl_0}(r_{bs}^2r_ye^{2ikl_y} - t_{bs}^2r_xe^{2ikl_x})} \tag{15}
$$

Solving for the reflected beam:

$$
\begin{aligned}
E_R &= r_mE_0 + it_mE_9 && \text{(16a)}\\
&= r_mE_0 + it_m(E_8e^{ikl_0}) && \text{(16b)}\\
&= r_mE_0 + it_me^{ikl_0}(r_{bs}E_7 + it_{bs}E_{12}) && \text{(16c)}\\
&= r_mE_0 + it_me^{ikl_0}(r_{bs}e^{ikl_y}E_6 + it_{bs}e^{ikl_x}E_{11}) && \text{(16d)}\\
&= r_mE_0 + it_me^{ikl_0}(r_{bs}r_ye^{ikl_y}E_5 + it_bsr_xe^{ikl_x}E_{10}) && \text{(16e)}\\
&= r_mE_0 + it_me^{ikl_0}(r_{bs}r_ye^{2ikl_y}E_3 + it_{bs}r_xe^{2ikl_x}E_4) && \text{(16f)}\\
&= r_mE_0 + it_me^{ikl_0}(r_{bs}^2r_ye^{2ikl_y}E_2 - t_{bs}^2r_xe^{2ikl_x}E_2) && \text{(16g)}\\
&= r_mE_0 + it_me^{2ikl_0}(r_{bs}^2r_ye^{2ikl_y} - t_{bs}^2r_xe^{2ikl_x})E_1 && \text{(16h)}
\end{aligned}
$$

$$
\frac{E_R}{E_0} = \frac{r_m - e^{2ikl_0}(r_m^2 + t_m^2)(r_{bs}^2r_ye^{2ikl_y} - t_{bs}^2r_xe^{2ikl_x})}{1 - r_me^{2ikl_0}(r_{bs}^2r_ye^{2ikl_y} - t_{bs}^2r_xe^{2ikl_x})} \tag{17}
$$

To find the power at the antisymmetric port as a fraction of the incident power on the cavity, we multiply equation 15 by its complex conjugate.

$$
P_T = \left[\frac{-t_me^{ikl_0}(t_{bs}r_{bs}r_xe^{2ikl_x} + t_{bs}r_{bs}r_ye^{2ikl_y})}{1 - r_me^{2ikl_0}(r_{bs}^2r_ye^{2ikl_y} - t_{bs}^2r_xe^{2ikl_x})}\right] \times \left[\frac{-t_me^{-ikl_0}(t_{bs}r_{bs}r_xe^{-2ikl_x} + t_{bs}r_{bs}r_ye^{-2ikl_y})}{1 - r_me^{-2ikl_0}(r_{bs}^2r_ye^{-2ikl_y} - t_{bs}^2r_xe^{-2ikl_x})}\right] \tag{18}
$$

To simplify, we will solve the numerator and denominator separately, and define the following variables: $l_y + l_o = l_y^{'}$, $l_x + l_o = l_x^{'}$, and $l_y + l_o = l_y^{'}$, and $l_y - l_x = l^{'}$. We will also use the following trigonometric identity: $cos(\phi) = \dfrac{e^{i\phi} + e^{-i\phi}}{2}$. Several intermediate steps have been omitted in the interest of saving space.

$$Numerator = (-t_m e^{ikl_0}(t_{bs}r_{bs}r_x e^{2ikl_x} + t_{bs}r_{bs}r_y e^{2ikl_y}))(-t_m e^{-ikl_0}(t_{bs}r_{bs}r_x e^{-2ikl_x} + t_{bs}r_{bs}r_y e^{-2ikl_y})) \tag{19a}$$

$$= t_m^2(t_{bs}^2 r_{bs}^2 r_x^2 + t_{bs}^2 r_{bs}^2 r_x r_y e^{2ik(l_x - l_y)} + t_{bs}^2 r_{bs}^2 r_x r_y e^{2ik(l_y - l_x)} + t_{bs}^2 r_{bs}^2 r_y^2) \tag{19b}$$

$$= t_m^2(t_{bs}^2 r_{bs}^2 r_x^2 + t_{bs}^2 r_{bs}^2 r_y^2 + 2t_{bs}^2 r_{bs}^2 r_x r_y cos(2kl^{'})) \tag{19c}$$

Similarly,

$$Denominator = (1 - r_m e^{2ikl_0}(r_{bs}^2 r_y e^{2ikl_y} - t_{bs}^2 r_x e^{2ikl_x}))(1 - r_m e^{-2ikl_0}(r_{bs}^2 r_y e^{-2ikl_y} - t_{bs}^2 r_x e^{-2ikl_x})) \tag{20a}$$

$$= 1 - 2r_{bs}^2 r_m r_y cos(2kl_y^{'}) + 2t_{bs}^2 r_m r_x cos(2kl_x^{'}) + r_m^2(r_{bs}^4 r_y^2 + t_{bs}^4 r_x^2 + 2r_{bs}^2 t_{bs}^2 r_x r_y cos(2kl^{'})) \tag{20b}$$

Therefore, the final expression for the power at the antisymmetric port is:

$$P_T = \frac{t_m^2 t_{bs}^2 r_{bs}^2 (r_x^2 + r_y^2 + 2r_x r_y cos(2kl^{'}))}{1 - 2r_m(r_{bs}^2 r_y cos(2kly') + t_{bs}^2 r_x cos(2kl_x^{'}) + r_m^2(r_{bs}^4 r_y^2 + t_{bs}^4 r_x^2 + 2r_{bs}^2 t_{bs}^2 r_x r_y cos(2kl'))} \tag{21}$$

References

[1] Anthony E. Siegman. *Lasers*. Sausalito, CA: University Science Books, 1986.

[2] Rainer Weiss, Concept for an interferometric test of Hogan's quantum geometry hypothesis. Unpublished thesis, Massachusetts Institute of Technology, 2009.

[3] Jacob D. Bekenstein. "Information in the holographic universe," *Scientific American*, 2003.

[4] Fermilab. 2009. "The Fermilab holometer: An experiment to measure the smallest intervals of space and time." [Online]. Available Web: http://holometer.fnal.gov/

[5] Fermilab. 2009. "The Fermilab holometer." [Online]. Available Web: http://holometer.fnal.gov/

[6] Craig Hogan (2010). "Holographic noise in Michelson interferometers." [Online]. Available Web: http://holometer.fnal.gov/presentations/hogansummary.pdf